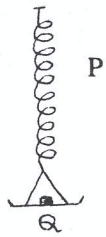
Eg(5): (1994 Q6)

A scale pan is suspended from a fixed point P by a light elastic spring. A particle Q of mass 0.2 kg is attached to the pan with glue. The pan is pulled down from its equilibrium position and set in motion. Given that the motion of Q is simple harmonic, with period $\frac{\pi}{6}$ seconds and that the maximum and minimum distances of Q below P are 1.5 m and 0.9 m, respectively, calculate



(i) the maximum speed of Q.
(ii) the maximum force that the glue has to exert on Q.
(iii) the length of the spring, when, in the absence of glue, Q would leave the pan.

Period $T = \frac{2\pi}{\omega} = \frac{\pi}{6}$

$\Rightarrow \quad \omega = 12$ rads/sec

Amplitude $= \frac{1.5 - 0.9}{2}$

$= \frac{.6}{2} = .3$

$\boxed{\omega = 12}$, $\boxed{A = .3}$

(i) Max speed of Q: where $x = 0$

$v^2 = \omega^2(A^2 - x^2) \Rightarrow v_{max} = \omega A.$

$\therefore \quad v_{max} = (12)(.3) = 3.6$ m/sec

(ii) Max force that the glue has to exert on Q is where the max acceleration takes place (ie at the highest point)

$|accel| = \omega^2 x.$

max $|Accel| = \omega^2 A$

max $|Accel| = (12)^2(.3)$

max $|accel| = 43.2$ (downwards)

Nett $F = ma \Rightarrow$ Max Nett Force $= .2(43.2)$

$\Rightarrow$ Max nett force $= 8.64$ N occurring at 0.9 below equilibrium

Forces on Q (typical position) (Not pan and Q)

NII $\Rightarrow \quad \Sigma f = ma$

$\Rightarrow .2g + f - R = .2a$

But at the position in trying to establish the max G has to be we assume $R = 0$ at top of motion

$\therefore \quad .2g + G = 8.64$

$1.96 + G = 8.64$

$\therefore$ max glue force $G = 6.68$ Newtons.

(iii) In the absence of glue Q will leave the pan where $R = 0$.

$\Sigma f = ma$

$\Rightarrow mg - R = m\omega^2 x$ (assuming SHM)

$\Rightarrow mg - 0 = m\omega^2 x$

$\Rightarrow \quad 9.8 = (12)^2 x$

$\Rightarrow \quad .068 = x$

Q leaves pan where pan is .068 m above the equilibrium position $\Rightarrow$

Length of spring $= 1.2 - .068 = 1.132$ m.